PROOF BY MATHEMATICAL INDUCTION

Most proofs by mathematical induction share several parts that have the same structure.

To prove a statement is true for all integers $n \geq 1$ (*):

1. Basis step: Prove the statement is true when $n = 1$ (*).

2. Inductive step:

   [a] Assume the statement is true for some particular but arbitrary integer $k \geq 1$ (*) (ie. when $n = k$).

   It is helpful to explicitly write down the statement when $n = k$, so you know what you're allowed to assume and use.

   [b] Prove the statement is true when $n = k + 1$.

   It is helpful to explicitly write down the statement when $n = k + 1$, so you know what you're trying to prove.

The proof in part 2[b] is different for each proof.
A frequent pattern of proving is to try to

[i] rewrite a complex expression from the step 2[b] so that the similar expression from step 2[a] appears
[ii] use the statement from 2[a] to make a statement from a slightly simpler expression
[iii] rewrite the slightly simpler expression so that the simpler expression from step 2[b] appears

(*) To prove a statement is true for all integers n ≥ some other number, replace these occurrences of 1 with that other number.

For each example below,

1. What are you supposed to prove is true in the basis step ?

2. [a] What are you supposed to assume is true in the inductive step ?

   [b] What are you supposed to prove is true in the inductive step ?

Example 1 $\dfrac{1}{1\times 4}+\dfrac{1}{4\times 7}+\dfrac{1}{7\times 10}+\cdots+\dfrac{1}{(3n-2)\times(3n+1)}=\dfrac{n}{3n+1}$

Example 2 $\sum_{i=1}^{n} i\cdot 2^{i} = (n-1)2^{n+1}+2$

Example 3 $\sum_{i=1}^{n} i\cdot i! = (n+1)!-1$

Example 4 $a_1+(a_1+d)+(a_1+2d)+\cdots+(a_1+(n-1)d)=\dfrac{n}{2}(2a_1+(n-1)d)$

Example 5 $1^2-2^2+3^2-4^2+\cdots+(-1)^{n+1}n^2=(-1)^{n+1}\dfrac{n(n+1)}{2}$